The 2023 World Congress on
Advances in Structural Engineering and Mechanics (ASEM23)
GECE, Seoul, Korea, August 16-18, 2023

Direct calculation of stress RAOs for floating structures

*Moonsu Park[1)], Phill-Seung Lee[2)]

[1), 2)] Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology
[1)] uhsa00@kaist.ac.kr
[2)] plee@kaist.ac.kr

ABSTRACT

This study introduces an efficient method for calculating the stress response amplitude operator (RAO) in frequency domain hydroelastic analysis. In frequency domain hydroelastic analysis, component stress responses are derived as a harmonic response and represented by real and imaginary parts. The ultimate strength and fatigue analysis of structures are evaluated using stress combined from these stress components, with von-Mises stress and principal stress being the main representatives. These stresses, calculated by combining component-wise stress, are no longer harmonic functions, but inharmonic functions. By introducing a method to directly calculate the RAO of these stresses in the frequency domain, we aim to propose a method that can be applied in key areas such as the design of ships and floating structures.

1. INTRODUCTION

The interest in the hydrodynamic analysis of various floating structures continues to rise due to the emergence of diverse floating structures. To evaluate the structural strength of a floating structure in waves, stress calculation is essential. Research on the hydroelastic analysis in the frequency domain, which has the advantage of fast calculation speed for finding various alternatives at the design stage, is developing in various forms(Kim et. al. 2013, Kim et. al. 2014, Yoon et. al. 2014, Lee et. al. 2015, Yoon et. al. 2017).

The actual sea consists of irregular waves, which can be expressed as a combination of

---

1) Graduate Student
2) Professor

responses to regular waves. The response to regular waves is represented by harmonic responses composed of real and imaginary parts, and the maximum response to a unit wave height regular wave is called RAO.

To evaluate the structural strength of a floating structure, combined stress, composed of the combination of component stresses, not the component stresses themselves, is used. Representative combined stresses include von Mises stress and principal stress. This study aims to introduce methods for calculating the RAOs of these combined stresses.

2. Hydroelastic equation

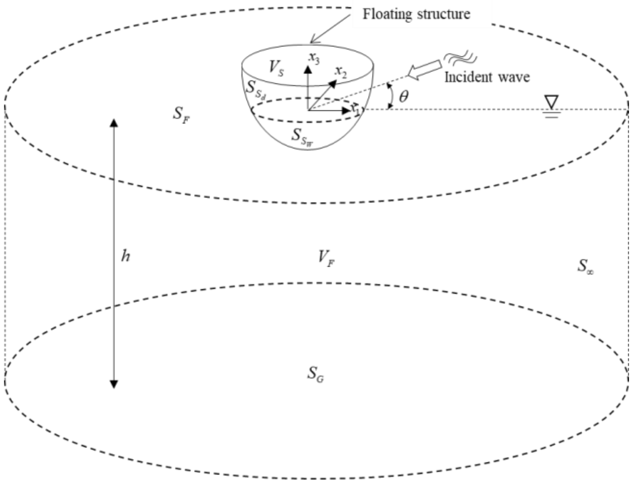


Fig. 1 Schematic of a floating structure

Fig. 1 represents the hydroelastic problem considered in this study. It is assumed that the floating structure has a homogeneous, isotropic, and linear elastic material and the fluid flow is incompressible, inviscid, and irrotational. The potential flow theory is employed. An incident regular wave comes from the positive x_1 direction with an angle θ and the amplitude is assumed to be small enough to use the linear wave theory.

The volume of the floating structure is denoted by V_s, and V_F represents the volume occupied by external fluid. The surface of the floating structure S_s is divided into dry and wet surfaces, S_{Sd} and S_{Sw}, respectively. The fluid domain is surrounded by the wet surfaces, free surface, the bottom surface, and the surface which is a circular cylinder with a sufficiently large radius R denoted by S_{Sw}, S_F, S_G, and S_∞, respectively.

For the problem, the following coupled equations are obtained (Kim et. al. 2013)

$$P_D = -j\omega\rho_w\phi, \tag{1}$$

$$\begin{aligned}&\omega^2\int_{{}^0V_s}{}^0\rho_s u_i\bar{u}_i dV+\int_{{}^0V_s}C_{ijkl}e_{kl}\,{}^0e_{ij}dV+\int_{{}^0V_s}{}^0\sigma_{ij}\bar{\eta}_{ij}dV\\&-\int_{{}^0S_{S_w}}\rho_w g u_3\,{}^0u_i\bar{u}_i dS-\int_{{}^0S_{S_w}}\rho_w g\,{}^0x_3\,{}^0n_j\mathrm{F}_{ij}\bar{u}_i dS-j\omega\int_{{}^0S_{S_w}}\rho_w\phi g\,{}^0n_j\bar{u}_i dS=0\end{aligned} \tag{2}$$

$$\alpha\int_{{}^0S_{S_w}}\phi\bar{\phi}\,dS-\int_{{}^0S_{S_w}}\left(\frac{\partial G}{\partial n_\xi}\phi-j\omega G u_i n_i\right)dS_\xi\bar{\phi}\,dS_x=4\pi\int_{{}^0S_{S_w}}\phi^I\bar{\phi}\,dS\,. \tag{3}$$

The finite element method is employed for the floating structure and the boundary element method is used for the external fluid. The discretized coupled equations for the steady state 3D hydro-elastic problems can be written as follow:

$$\begin{bmatrix}-\omega^2\mathbf{S}_M+\mathbf{S}_K+\mathbf{S}_{KN}-\mathbf{S}_{HD}-\mathbf{S}_{HN} & j\omega\mathbf{S}_D\\ j\omega\mathbf{F}_G & \alpha\mathbf{F}_M-\mathbf{F}_{Gn}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{u}}\\ \hat{\phi}\end{bmatrix}=\begin{bmatrix}0\\ 4\pi\mathbf{R}_I\end{bmatrix}. \tag{4}$$

3. Stress response

The displacement responses obtained from Eq. (4) can be expressed in the form of complex numbers or trigonometric functions. When represented in complex number form, it can be expressed as real and imaginary components, or in general trigonometric function form as the following equation, assuming the specific frequency ω .

$$\hat{u}_i=\hat{u}_i^{\mathrm{Re}}+\hat{j}\hat{u}_i^{\mathrm{Im}} \tag{5}$$

$$u_i(t)=\mathrm{Re}\{\hat{u}_i\,e^{\hat{j}\omega t}\}=\hat{u}_i^{\mathrm{Re}}\cos\omega t-\hat{u}_i^{\mathrm{Im}}\sin\omega t\,, \tag{6}$$

in which

$$\hat{u}_i\,e^{\hat{j}\omega t}=\hat{u}_i\cos\omega t-\hat{j}\hat{u}_i\sin\omega t\,.$$

For the stress analysis,

$$\hat{\sigma}_{ij}=C_{ijrs}\hat{\varepsilon}_{rs}=\hat{\sigma}_{ij}^{\mathrm{Re}}+\hat{j}\hat{\sigma}_{ij}^{\mathrm{Im}}\,, \tag{7}$$

where,

$$\hat{\sigma}_{ij}^{\mathrm{Re}}=C_{ijrs}\hat{\varepsilon}_{rs}^{\mathrm{Re}}\,,\ \hat{\sigma}_{ij}^{\mathrm{Im}}=C_{ijrs}\hat{\varepsilon}_{rs}^{\mathrm{Im}}\,.$$

In the frequency domain, the components of stress, similar to displacement, are also expressed in the form of harmonic response.

$$\sigma_{ij}(t)=\hat{\sigma}_{ij}^{\mathrm{Re}}\cos\omega t-\hat{\sigma}_{ij}^{\mathrm{Im}}\sin\omega t\,, \tag{8}$$

In order to evaluate the yield and fatigue strength of a structure, a combined stress is used. Representative combined stresses include von-Mises stress and principal stress.

The von-Mises stress is a representative stress for evaluating the yield stress, and is a stress using the second-order deviation stress invariant. The equation for obtaining the von-Mises stress in the time domain is as follows, and the coefficient terms are organized by the following equations

$$\sigma_{vM}(t) = \sqrt{\frac{3}{2}\left(\sigma_{ij}(t) - \frac{1}{3}\delta_{ij}\sigma_{kk}(t)\right)^2}\,. \tag{9}$$

The concept of principal stress is commonly used in engineering and mechanics to analyze the failure or deformation behavior of materials and structures, such as in structural engineering, geotechnical engineering, and solid mechanics. Because crack growth is closely related to the angle of principal stress, principal stress is often used for fatigue analysis. The principal stress in a three-dimensional stress state can be defined by the stress invariants (I_1, I_2, I_3) as follows.

$$\begin{aligned}
P_1 &= \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\phi\,,\\
P_2 &= \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\left(\phi(t) - \frac{2\pi}{3}\right),\\
P_3(t) &= \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\left(\phi(t) - \frac{4\pi}{3}\right),
\end{aligned} \tag{10}$$

where,

$$I_1(t) = \sigma_{ii}(t)\,,$$

$$I_2(t) = \frac{1}{2}\left\{\sigma_{ii}(t)\sigma_{jj}(t) - \sigma_{ij}(t)\sigma_{ji}(t)\right\},$$

$$I_3(t) = \varepsilon_{ijk}\sigma_{1i}(t)\sigma_{2j}(t)\sigma_{3k}(t)\,,$$

4. Direct calculation of stress response

The Response Amplitude Operators (RAOs) for individual components of the wave, given a particular frequency, can be easily determined from the magnitude of the real and imaginary parts as they are harmonic responses. However, the calculation of the combined RAO is not straightforward since it is a non-harmonic function. The current methodology we employ is depicted in Fig. 2. To identify the peak values of von-Mises and principal stresses over a single cycle, we divide the cycle into 36 equal intervals and use Eq. (8) to derive the component stress values for the corresponding times. We then combine these component stresses to compute the combined stress and use the maximum value as the RAO. In this study, we aim to introduce an efficient method for calculating the combined stress RAO.

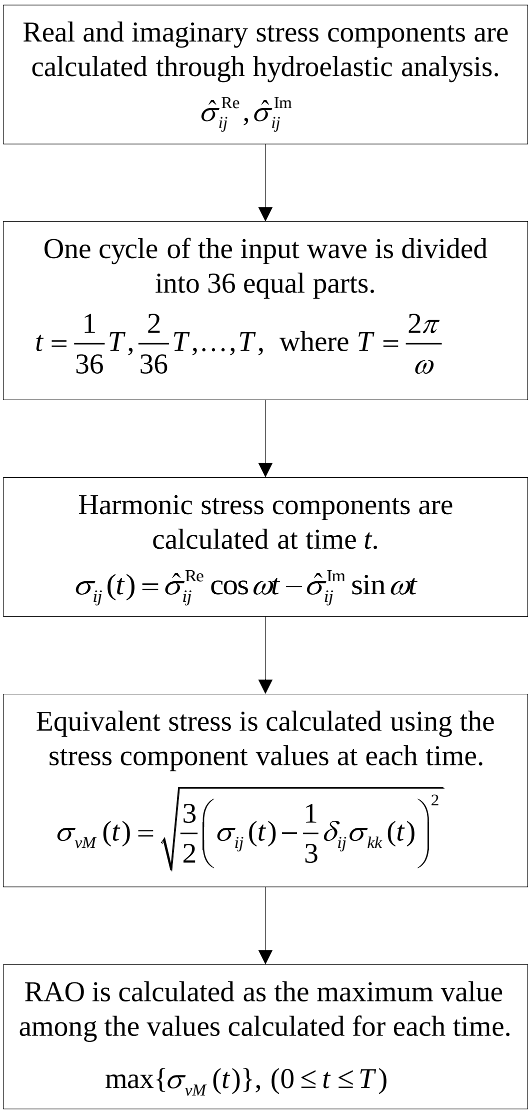


Fig. 2 Conventional stress RAO calculation procedure

In the case of von-Mises stress, if we substitute Eq. (8) into Eq. (9) and rearrange, we can obtain the following equation.

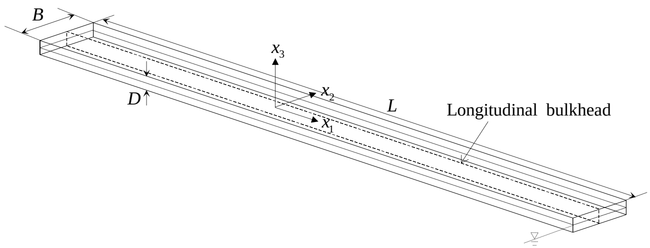
$$\sigma_{vM}(t) = \sqrt{\frac{\sqrt{(A-B)^2 + C^2}}{2} \sin(2\omega t + \phi_1) + \frac{A+B}{2}} \,. \tag{11}$$

The maximum value of Eq. (11) and its corresponding time can be determined by organizing the coefficients as follows.

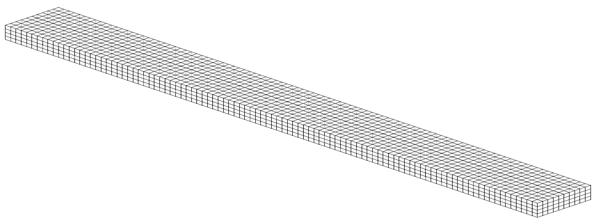
$$\max(\sigma_{vM}) = \sqrt{\frac{\sqrt{(A-B)^2 + C^2}}{2} + \frac{A+B}{2}} \quad \text{when } 0 \le \omega t = n\pi + \frac{\pi}{4} - \frac{\phi_1}{2} \le 2\pi . \tag{12}$$

For principal stress, it can be obtained by substituting Eq. (8) into Eq. (10). Given that it consists of the sum of harmonic and non-harmonic functions, analytically determining the maximum value is not an easy task. We utilize the Newton-Raphson method to find the solution, seeking the value where the first derivative equals zero, and find the maximum value at that position.

5. Numerical example



(a)



(b)

Fig. 3 Floating barge problem: (a) problem description and (b) finite and boundary element meshes used.

Let us consider the floating barge problem shown in Fig.3. The barge's dimensions are the same with the one used in the reference [59]; that is, the length *L* is 100 m, the breadth *B* is 10 m, and the depth *D* is 2 m. A longitudinal bulkhead is additionally installed along the centerline in the barge model.

For the barge model, 100, 10, and 4 shell finite elements(Lee et. al. 2004, Lee et. al. 2012, Jeon et. al. 2014, Lee et. al. 2014, Jeon et. al. 2015). are used in the length, breadth, and depth directions, respectively, and 100, 10, and 2 boundary elements are used for the fluid interface, respectively. We use the elastic modulus $E = 100$ GPa, Poisson's ratio $\upsilon = 0.3$, wave period $T = 4-16s$, and incident wave angle $\theta = 0°$. Then, hydroelastic analyses are carried out through Eq. [4]. von-Mises stress and principal stress RAO are obtained using the calculated component stresses from hydroelastic analysis.

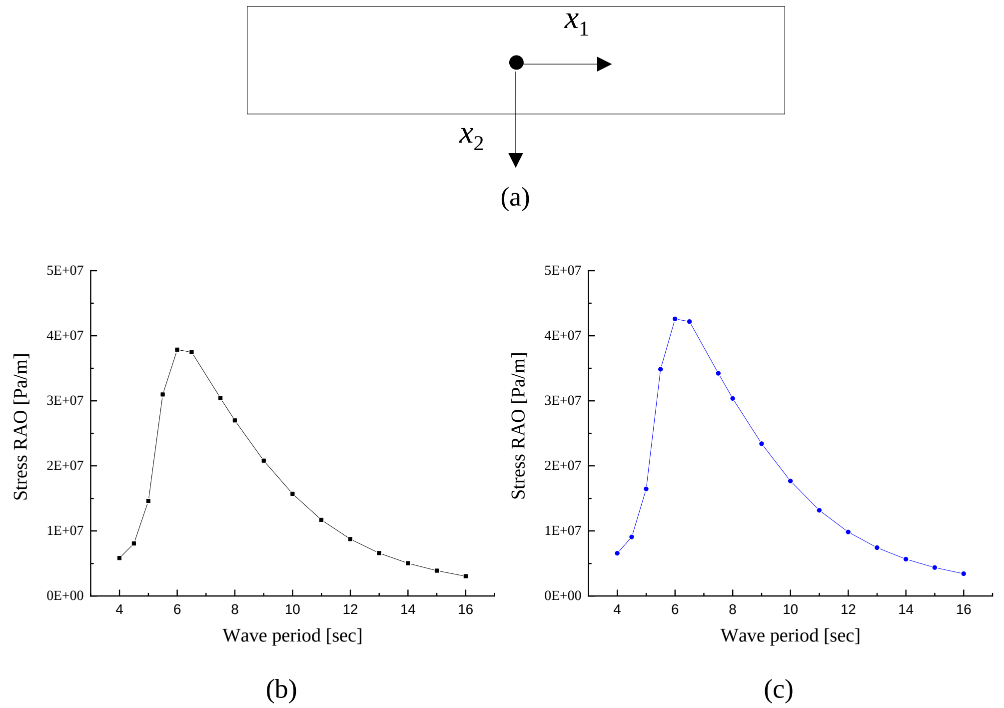


Fig.4 Stress RAO of the floating barge
: (a) measuring point, (b) von-Mises stress RAO and (c) principal stress RAO.

Table 1. Computational time of stress RAO Calculation in floating barge

	von-Mises stress RAO		Principal stress RAO	
	Previous	Proposed	Previous	Proposed
Computational time [sec]	19.36	1.47 (7.6%)	19.78	5.42 (27.4%)

Fig.4 shows the stress RAO using the proposed method. The RAO is measured the center of the bottom of the barge. Table 1 compares the computation times for stress RAO between the proposed method and the conventional approach. For von-Mises stress, we can observe a 7.5% reduction in computation time, while for the principal stress, we see a decrease of 27.4% in computation time.

6. Conclusions

We proposed a direct calculation method for the stress response amplitude operator (RAO) in the frequency domain for hydroelastic analysis. After calculating the component stresses using hydroelastic analysis and evaluating the strength using combined stresses such as von-Mises stress and principal stresses, the combined stresses are no longer in a harmonic form. Therefore, instead of using the conventional method, we propose a direct method to find the maximum value. This approach allows for significant improvement in computational speed. We believe that our proposed method can contribute to the enhancement of computational efficiency in various applications such as ship and offshore structure design.

Acknowledgments
This research was supported by the Challengeable Future Defense Technology Research and Development Program (No.915071101) of Agency for Defense Development in 2023.

REFERENCES

Kim KT, Lee PS, Park KC, A direct coupling method for 3D hydroelastic analysis of floating structures. Int J Numer Methods Eng. 2013, 842-866.

Lee KH, Lee PS, Nonlinear hydrostatic analysis of flexible floating structures. Appl Ocean Res. 59 (2016), 165-182.

Lee KH, Cho S, Kim KT, Kim JG, Lee PS, Hydroelastic analysis of floating structures with liquid tanks and comparison with experimental tests. Appl Ocean Res. 52(2015), 167-187.

Yoon JS, Lee PS, Towards hydro-elastoplastic analysis of floating plate structures. J Fluids Struct. 71(2017), 164-182.

Kim JG, Cho SP, Kim KT, Lee PS, Hydroelastic design contour for the preliminary design of very large floating structures. Ocean Eng.,78 (2014), 112-123.

Yoon, J. S., Cho, S. P., Jiwinangun, R. G., Lee, P. S., Hydroelastic analysis of floating plates with multiple hinge connections in regular waves. Marine structures, 36 (2014), 65-87.

Lee PS, Bathe KJ. Development of MITC isotropic triangular shell finite elements, Computers and Structures, 82(11-12), (2004), 945-962.

Lee YG, Yoon KH, Lee PS. Improving the MITC3 shell finite element by using the Hellinger-Reissner principle, Computers and Structures (2012), 110-111, 93-106.

Jeon HM, Lee Y, Lee PS, Bathe KJ. The MITC3+ shell element in geometric nonlinear analysis, Computers and Structures, 146 (2015), 91-104.

Lee Y, Lee PS, Bathe KJ. The MITC3+ shell finite element and its performance, Computers and Structures, 138 (2014), 12-23.

Jeon HM, Lee PS, Bathe KJ. The MITC3 shell finite element enriched by interpolation covers, Computers and Structures, 134(2014), 128-142.